Math 70 5.9 Algebra of functions Lesson 17
6.1 Intro to Factoring
6.2 Factoring unitary trinomials

5.9 Algebra of Functions

Objectives

1) Write the sum, difference, product, or quotient of two functions
2) Understand notation

* We will not do domain or range *

6.1 Introduction to Polynomial Factorizations & Equations

Objectives

1) Review solving by graphing
   - intersect
   - x-intercept
2) Use the Zero-product property to solve equations
3) Factor the greatest common factor
4) Factor by grouping

6.2 Factoring Trinomials x^2+bx+c

1) Factor trinomials of degree 2 with leading coefficient 1.
2) Factor out GCF -1 when leading coefficient is -1.
3) Solve equations containing $x^2+bx+c=0$.

Math 70 5.9 Algebra of Functions

Objectives

1) Find a new function which is the sum $(f+g)(x)$, difference $(f-g)(x)$, product $(f \cdot g)(x)$, or quotient $(f/g)(x)$ of two given functions.
2) Recognize function notation and notation for the names of these functions.

Practice and Examples

1) Given $f(x)=3x^2+4x+1$ and $g(x)=2x-5$, find:

a) $(f+g)(x)$ b) $(f-g)(x)$ c) $(f \cdot g)(x)$ d) $(f/g)(x)$

2) Given $\begin{cases} f(-1)=4 & g(-1)=-4 \\ f(0)=5 & g(0)=-3 \\ f(2)=7 & g(2)=-1 \\ f(7)=1 & g(7)=9 \end{cases}$, find

a) $(f+g)(2)$ c) $(f \cdot g)(7)$ e) $(f/g)(0)$

b) $(f-g)(0)$ d) $(f \cdot g)(0)$ f) $(g/f)(0)$

Math 70: 6.1 & 6.2 Factoring Expressions (GCF & Grouping) and Solving Equations

Objectives
1) Solve nonlinear equations using graphing methods or factoring with zero-product property.
2) Factor expressions containing monomial Greatest Common Factor (GCF)
3) Factor expressions containing binomial Greatest Common Factor (GCF)
4) Factor expressions with negative leading coefficients
5) Factor expressions with four terms using grouping (GCF three times)
6) Factor expressions with three terms and unitary leading coefficient
7) Factor expressions requiring two or more of these skills

Solve by graphing (both intersection and x-intercept methods) then by factoring with the zero-product property.

1) $x^2 = 6x$

Write an equivalent expression by factoring.

2) $-8p^6q^2 + 4p^5q^3 - 10p^4q^4$

3) $15x^5 - 12x^4 + 27x^3 - 3x^2$

4) $7x(x^2 + 5y) + 3a(x^2 + 5y)$

5) $(a-b)(x+5) + (a-b)(x-y^2)$

6) $x^3 + 3x^2 - 5x - 15$

7) $4t^3 - 15 + 20t^2 - 3t$

8) $t^2 - 9t + 20$

9) $x^2 - 48y^2 + 2xy$

10) $x^2 + x - 7$

Find the zeros of the function.

11) $h(t) = -16t^2 + 64t$

Find all the values of *a* for which $f(a) = 0$

12) $f(x) = x^3 - 3x^2 + 4x - 12$

Solve.

13) $-2x^3 + 26x^2 = -1216x$

14) $(t-10)(t+1) = -24$

Math 70 B. 5.9 Algebra of Functions

① Given $f(x) = 3x^2+4x+1$
$g(x) = 2x-5$

a) Find $(f+g)(x)$ ← Say this "f plus g of x"
Name of function: $f+g$
variable used: x
NOT multiply by x.

$= f(x) + g(x)$
$= 3x^2+4x+1 + 2x - 5$ — add the two functions, combine like terms
$= \boxed{3x^2+6x-4}$

b) $(f-g)(x)$ ← Say: "f minus g of x"
Name of function: $f-g$
Variable used: x
NOT multiply by x

$= f(x) - g(x)$
$= (3x^2+4x+1) - (2x-5)$ — use ( ) to dist neg.
$= 3x^2+4x+1 - 2x + 5$ — dist neg
$= \boxed{3x^2+2x+6}$ — combine like terms

c) $(f\cdot g)(x)$ ← Say: "f times g of x"
CAUTION: • Not $\circ$
{ $\circ$ means something else, in chap 12}
must use ( )

$= f(x)\cdot g(x)$
$= (3x^2+4x+1)(2x-5)$
$= 6x^3 - 15x^2$
$\quad + 8x^2 - 20x$
$\quad + 2x - 5$ — mult by ~~list~~ dist $3x^2$, $4x$, 1

$= \boxed{6x^3 - 7x^2 - 18x - 5}$ — combine like terms.

d) $(f/g)(x)$ ← Say "f divided by g of x"
$= \dfrac{f(x)}{g(x)}$
$= \boxed{\dfrac{3x^2+4x+1}{2x-5}}$ — factor + cancel if possible

Math 70

② Given $f(-1)=4$ $\quad g(-1)=-4$

$f(0)=5$ $\quad g(0)=-3$

$f(2)=7$ $\quad g(2)=-1$

$f(7)=1$ $\quad g(7)=9$

find

a) $(f+g)(2)$ ← Say: "f plus g of 2"

$= f(2)+g(2)$

$= 7+(-1)$ subst given values

$= \boxed{6}$

b) $(f-g)(0)$

$= f(0)-g(0)$

$= 5-(-3)$ subst given values

$= 5+3$

$= \boxed{8}$

c) $(f\cdot g)(7)$

$= f(7)\cdot g(7)$

$= 1\cdot 9$

$= \boxed{9}$

d) $(f\cdot g)(0)$

$= f(0)\cdot g(0)$

$= 5\cdot(-3)$

$= \boxed{-15}$

e) $(f/g)(0)$

$= \dfrac{f(0)}{g(0)}$

$= \dfrac{5}{-3} \doteq \boxed{\dfrac{-3}{5}}$

f) $(g/f)(0)$

$= \dfrac{g(0)}{f(0)}$

$= \boxed{\dfrac{-3}{5}}$

Math 70 B. 6.1 & 6.2

① Solve by both graphing methods and zero product property.

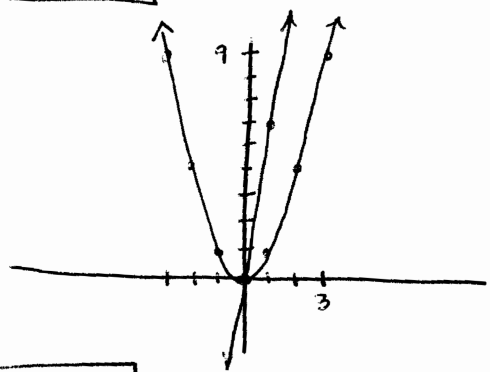
* Solving by different methods should always give the same resu

Method 1: Graphing $x^2 = 6x$ using intersections

GC: $\boxed{y=}$ $y_1 = x^2$ left side of =

$y_2 = 6x$ right side of =

$\boxed{\text{ZOOM}}$ 6. standard

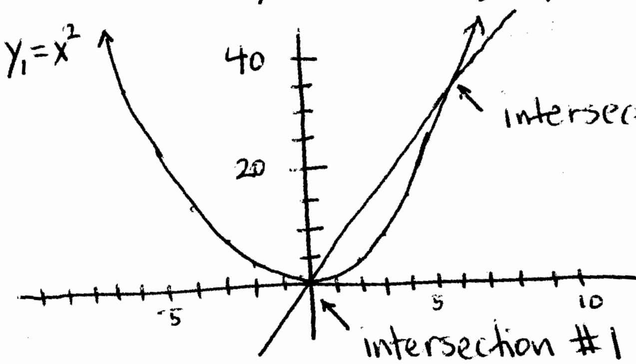


We can see one point of intersection (the origin (0,0))

but not the other

Need more y-values at top of graph.

$\boxed{\text{WINDOW}}$ ymax = 40, yscl = 5



$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ (CALC) 5. Intersect

→ (0,0)

Repeat

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ (CALC) 5. Intersect

* Must move cursor! * ▶ → (6,36)

solutions are x-coordinates only.

$\boxed{x = 0, 6}$

Math 70 6.1 & 6.2

Method 2: Graphing using x-intercepts.

$x^2 = 6x$

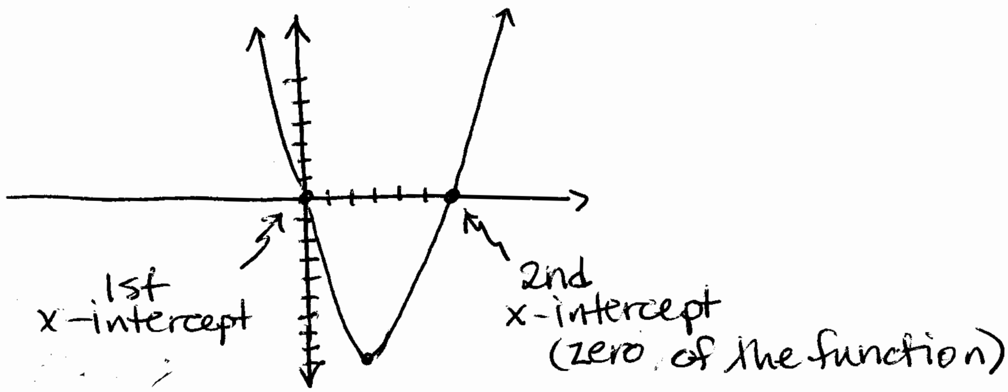
set $= 0$ by subtracting $6x$ both sides

$x^2 - 6x = 0$

GC : $\boxed{Y=}$ $y_1 = x^2 - 6x$

$y_2 = \boxed{\text{CLEAR}}$

$\boxed{\text{ZOOM}}$ 6. standard



CALC

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ 2: zero $\longrightarrow$ $(0,0)$

Repeat CALC

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ 2. zero $\longrightarrow$ $(6,0)$

solutions are x-intercepts only.

$\boxed{x = 0, 6}$

Method 3: Zero product property

Step 1: set equal to zero

$x^2 = 6x$

$x^2 - 6x = 0$

step 2: Factor out the GCF = divide both terms by x

$x\left(\frac{x^2}{x} - \frac{6x}{x}\right) = 0$ ← most people do this step in their heads.

Math 70 6.1 & 6.2

$$x(x-6)=0$$

step 3: Set each factor $=0$.

$x=0 \qquad x-6=0$

step 4: isolate variable in each result.

$\boxed{x=0} \qquad \boxed{x=6}$

Write an equivalent expression by factoring.

② $\underbrace{-8p^6q^2}_{1st} + \underbrace{4p^5q^3}_{2nd} - \underbrace{10p^4q^4}_{3rd}$

step 1: count the terms, separated by + or −, ignoring any + or − at the start or inside ().

There are 3 terms.

step 2: Identify the sign of the common factor. If the first (leading) term is negative, the GCF is neg.

Step 3: Identify the largest number that will divide into all three coefficients $\frac{8}{2}$ $\frac{4}{2}$ $\frac{10}{2}$ evenly

but nothing larger will divide evenly.

step 4: Consider any variable which is in all terms, choose smallest exponent $p^6,\ p^5,\ p^4 \longrightarrow p^4$

$q^2\ \ q^3\ \ q^4 \longrightarrow q^2$

step 5: Determine GCF as product of steps 2-3-4

$-2p^4q^2$

step 6: Factor out GCF $\quad -2p^4q^2\left(\frac{-8p^6q^2}{-2p^4q^2}+\frac{4p^5q^3}{-2p^4q^2}-\frac{10p^4q^4}{-2p^4q^2}\right)$

$$= -2p^4q^2(4p^2-2pq+5q^2)$$

does $(4p^2-2pq+5q^2)$ factor? Tune in for 6.3!!

Math 70 6.1 & 6.2

(3) $\underbrace{15x^5}_{\text{1st term}} - \underbrace{12x^4}_{\text{2nd}} + \underbrace{27x^3}_{\text{3rd}} - \underbrace{3x^2}_{\text{4th}}$

15 (+)

$\frac{15}{3}$ $\frac{-12}{3}$ $\frac{27}{3}$ $\frac{-3}{3}$

x^5 x^4 x^3 $x^2 \rightarrow x^2$

$$= 3x^2\left(\frac{15x^5}{3x^2} - \frac{12x^4}{3x^2} + \frac{27x^3}{3x^2} - \frac{3x^2}{3x^2}\right)$$

$$= \boxed{3x^2(5x^3 - 4x^2 + 9x - 1)}$$

All factoring problems can be checked using multiply (in this case distribute $3x^2$)
If factoring is done correctly, we should get original

$$3x^2 \cdot 5x^3 - 3x^2 \cdot 4x^2 + 3x^2 \cdot 9x - 3x^2 \cdot 1$$
$$= 15x^5 - 12x^4 + 27x^3 - 3x^2 \checkmark$$

(4) $\underbrace{7x(x^2+5y)}_{\text{1st}} + \underbrace{3a(x^2+5y)}_{\text{2nd}}$

Remember: + or − inside ( ) don't separate terms

each term has (x^2+5y) = binomial GCF

$$= (x^2+5y)\left(\frac{7x\cancel{(x^2+5y)}}{\cancel{(x^2+5y)}} + \frac{3a\cancel{(x^2+5y)}}{\cancel{(x^2+5y)}}\right)$$

$$= \boxed{(x^2+5y)(7x+3a)}$$

Math 70 6.1 & 6.2

⑤ $\underbrace{(a-b)(x+5)}_{1st} + \underbrace{(a-b)(x-y^2)}_{2nd}$ Both terms have $(a-b)$

$= (a-b)\left(\frac{\cancel{(a-b)}(x+5)}{\cancel{(a-b)}} + \frac{\cancel{(a-b)}(x-y^2)}{\cancel{(a-b)}}\right)$

$= (a-b)\left((x+5) + (x-y^2)\right)$

These innermost ( ) no longer have meaning.

$= (a-b)(x+5+x-y^2)$

combine like terms $x+x=2x$

$= \boxed{(a-b)(2x+5-y^2)}$

⑥ $x^3+3x^2-5x-15.$ 4 terms, no GCF

{1st term has coef 1
{last term has no x

Factor by grouping.

$= \underbrace{x^3+3x^2}_{\text{group \#1}} + \underbrace{-5x-15.}_{\text{group \#2}}$ change group #2 to an addition

group #1 GCF x^2

group #2 GCF $= -5$

$= x^2(x+3)+(-5)(x+3)$ Check! Grouping GCF #3 must be the same

$= \underbrace{x^2(x+3)}_{\text{1st term}} - \underbrace{5(x+3)}_{\text{2nd term}}$ The expression now has only two terms

Both have binomial GCF $(x+3)$ = GCF #3

$= (x+3)\left[\frac{x^2\cancel{(x+3)}}{\cancel{(x+3)}} - \frac{5\cancel{(x+3)}}{\cancel{(x+3)}}\right]$

$= \boxed{(x+3)(x^2-5)}$ Does x^2-5 factor? Tune in for 6.4 !!

Math 70 6.1 & 6.2

⑦ $4t^3 - 15 + 20t^2 - 3t$

1st 2nd 3rd 4th

Notice 4 terms
Not in standard form.
No GCF $\rightarrow$ coefficient 3 & 4
$\rightarrow$ no variable -15

<u>Method 1</u>: Grouping in the order given

$$\underbrace{4t^3 - 15}_{\text{no GCF}} + \underbrace{20t^2 - 3t}_{\text{GCF } t}$$

$$= 1(4t^3 - 15) + t(20t - 3)$$

(stuff) is not a common factor because it's different. ☹

This approach failed. ☹

<u>Method 2</u>: Write terms in standard form first (highest to lowest exponent/degree)

$$= \underbrace{4t^3 + 20t^2}_{\text{GCF } 4t^2} - \underbrace{3t - 15}_{\text{GCF } -3}$$

$$= \underbrace{4t^2(t+5)}_{\text{1st}} - \underbrace{3(t+5)}_{\text{2nd}}$$

← Both terms have GCF $(t+5)$

$$= \boxed{(t+5)(4t^2-3)}$$

← Does $(4t^2-3)$ factor? Tune in for 6.4!

⑧ $t^2 - 9t + 20$

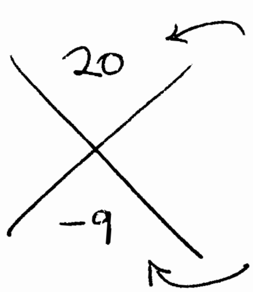
1st 2nd 3rd

called a <u>trinomial</u>

This expression has only 3 terms, cannot be grouped.

t^2 is the leading term, its coefficient is 1. (unitary)
GOAL: find two binomial factors $(\odot \pm \odot)(\odot \pm \odot)$ which FOIL to give $t^2 - 9t + 20$.

Magic X:



$1x^2 + bx + \underline{c}$

write c at top including any negatives

write b at bottom including any negatives.

Math 70 6.1 & 6.2

we want two numbers which multiply to +20 but add to −9.

Factors of 20	1×20	or	-1×-20
	2×10	or	-2×-10
	4×5	or	-4×-5

In order to add to a negative number, both factors must be negative. Only −4 and −5 work.

20
−4 −5 ← completed "MagicX".
−9

$$\boxed{(t-4)(t-5)}$$

Check by multiplying using the FOIL method

$(t-4)(t-5)$

$= t^2 - 5t - 4t + 20$

$= t^2 - 9t + 20$ ✓

⑨ $\underbrace{x^2}_{1st} \underbrace{-48y^2}_{2nd} \underbrace{+2xy}_{3rd}$

3 terms = trinomial

* These terms are not in a useful form — choose one variable and arrange by descending exponents of that variable.

$= x^2 + 2xy - 48y^2$

$x^2 \to x^1 \to x^0$

$y^0 \to y \to y^2$

⇒ That variable will appear in the first term of each binomial factor:

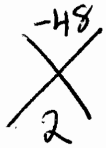
$(x \quad)(x \quad)$

A variable with ascending exponents will appear in the <u>second</u> term of each binomial factor

$(x \quad y)(x \quad y)$

math 70 6.1 & 6.2

Find coefficients using the magic X method



factors that multiply to -48

$1 \times (-48)$ or $(-1) \times 48$

$2 \times (-24)$ or $(-2) \times 24$

$3 \times (-16)$ or $(-3) \times 16$

$4 \times (-12)$ or $(-4) \times 12$

$6 \times (-8)$ or $(-6) \times 8$

$\boxed{(x-6y)(x+8y)}$

check by FOIL: $x^2+8xy-6xy-48y^2$

$= x^2+2xy-48y^2$ ✓

(10) x^2+x-7

3 terms

magic X

numbers that multiply to -7

$1 \times (-7)$ or $(-1) \times 7$

no others, none work

$\boxed{\text{prime}}$

which an expression cannot be factored by any factoring method, we say it is prime.

(11) Find the zeros of the function $h(t) = -16t^2+64t$

set $h(t)=0$ $\quad -16t^2+64t=0$

factor GCF, with negative: $\quad -16t(t-4)=0$

Set factors $=0$ $\quad -16t=0 \quad t-4=0$

isolate variable $\quad t=\frac{0}{-16} \quad \boxed{t=4}$

$\boxed{t=0}$

Math 70 6.1 & 6.2

(12) Find all the values of a for which $f(a)=0$ when $f(x) = x^3 - 3x^2 + 4x - 12$

evaluate when x is replaced by a

$$f(a) = a^3 - 3a^2 + 4a - 12$$

set $=0$

$$a^3 - 3a^2 + 4a - 12 = 0$$

count 4 terms $\Rightarrow$ factor by grouping

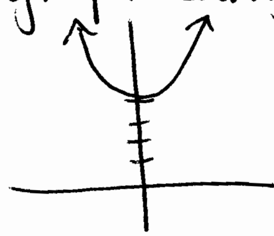
$$a^2(a-3) + 4(a-3) = 0$$

$$(a-3)(a^2+4) = 0$$

Set equal to 0

$a - 3 = 0$ $\qquad$ $a^2 + 4 = 0$

$\boxed{a=3}$ $\qquad$ ↖ solve graphically (for now)



does not cross x-axis $\Rightarrow$ has no real solutions

Tune in chapter 11 for non-real solutions!

Solve

(13) $-2x^3 + 26x^2 = -1216x$

set $=0$ $\quad 2x^3 - 26x^2 - 1216x = 0$

factor GCF $2x$

$$2x(x^2 - 13x - 608) = 0$$

factor trinomial

-608

19 $\quad$ -32

-13

1×-608
2×-304
4×-152
8×-76
$16 \times -38 \rightarrow -22$
$19 \times -32 \rightarrow -13$ ☺

$$2x(x+19)(x-32) = 0$$

set factors $= 0$

$2x = 0 \qquad x + 19 = 0 \qquad x - 32 = 0$

isolate

$\boxed{x=0 \qquad x=-19 \qquad x=32}$

Math 70 6.1 & 6.2

(14) $(t-10)(t+1) = -24$

The zero product property works <u>ONLY</u> when right side is zero.

$$\underbrace{(t-10)(t+1)}_{\text{1st}} + \underbrace{24}_{\text{2nd}} = 0$$

not factored, and not GCF. FOIL first term and start over with factoring process.

$t^2 - 9t - 10 + 24 = 0$

combine like terms

$t^2 - 9t + 14 = 0$

Now we have 3 terms, a magic X

$(t-7)(t-2) = 0$

$\boxed{t=7}$ $\boxed{t=2}$

14

$-7 \quad \times \quad -2$

-9

14×1 $\quad$ $-14 \times (-1)$

7×2 $\quad$ $-7 \times (-2)$